BEBR

FACULTY WORKING PAPER NO. 1171

Preliminary Empirical Evidence Regarding the Pricing of Estimation Risk

David A. Ziebart

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign

BEBR

FACULTY WORKING PAPER NO. 1171

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

August, 1985

Preliminary Empirical Evidence Regarding
The Pricing of Estimation Risk

David A. Ziebart, Assistant Professor
Department of Accountancy

First Draft

Comments and criticisms are welcome - please do not quote.

Abstract

The traditional approach to optimal portfolio selection assumes that the estimated parameters are known with certainty. However, the existence of this estimation risk has been documented in the literature. Assuming diversification of unsystematic risk, this study hypothesizes that estimation risk regarding the beta of a portfolio is priced in an equilibrium market. Specifically, this paper empirically tests for the presence of higher returns on portfolios in which the estimate of beta is less certain and lower returns on portfolios in which the estimate of beta is more certain. The results are supportive of the hypothesis.

Introduction

In most investment contexts an individual has only imperfect knowledge of the values of the parameters employed in the capital asset pricing model for either individual securities or portfolios. Given this imperfect knowledge, the issue of estimation risk becomes important in the choice of securities as an investment. Bawa, Brown, and Klein (1979) demonstrate analytically that estimation risk leads to optimal portfolio choices which are different than those found when the parameters are treated as certain. This paper is an empirical assessment of the pricing by the market of this additional risk component.

The next section of this paper summarizes the argument linking imperfect knowledge of investment parameters to estimation risk and market returns. A description of the empirical anlysis and the results is provided in the third section. Section four summarizes the findings and implications of this study.

Estimation Risk and Optimal Portfolio Choice

The traditional approach to optimal portfolio selection assumes that the estimated parameter values are known with certainty and any risk associated with the parameters being uncertain is ignored. The existence of this estimation or measurement risk has been documented in both the portfolio theory and the capital market equilibrium literature. Markowitz (1952) recognized the problem and suggested that the mean-variance approach incorporate a probabalistic approach to the parameter uncertainty problem. Joyce and Vogel (1970), Frankfurter, Phillips and Seagle (1971), Dickinson

(1974), and Miller and Scholes (1972) all point out problems associated with the use of sample estimates as surrogates for the true parameters. Yet, given this knowledge of the problems associated with estimation risk, the common approach in investments is to use the sample estimate and disregard any uncertainty.

The Sharp (1963) index model postulates that the returns of securities or portfolios are related to each other through their common relationship to an index of market activity. The asset pricing model (Sharp 1964, Litner 1965) implies that the intercept and the beta terms are the essence of pricing behavior. In most theoretical discussions of this model the investor is assumed to know the parameter values with certainty. Practically, however, investors do not know the true parameter values; the parameters are usually estimated by regressing the security or portfolio returns on the returns of a market index using ordinary least squares regression. However, implicit in the application of regression to estimate the model parameters is the sampling distribution of the estimated coefficient. Estimates of the coefficients are obtained through the application of regression but the estimates are still subject to uncertainty.

In a diversified portfolio the key element is the systematic risk coefficient; unsystematic risk is diversified away. The beta of the portfolio reflects the sensitivity of the return on the portfolio to the returns of the market as a whole. However, in applications of modern portfolio theory the beta coefficient is estimated and is subject to a sampling distribution. This sampling distribution which results from the estimation process is the estimation risk regarding the beta coefficient. Given an equilibrium market, this estimation risk should be priced. By appealing to an arbitrage argument it is apparent that, given all other

things being equal, the presence of uncertainty regarding the beta estimate should be priced. Assume that for two securities or portfolios one obtains beta estimates of 1.00 for both with a standard error of estimate for one of .05 and .10 for the other. Given equal levels of confidence the confidence interval around the beta estimate is twice as large for the large standard error security as for the small standard error security. The confidence interval is .90 to 1.10 for the small standard error security and .80 to 1.20 for the large standard error security when a 95% level is employed. Given this difference in estimation risk the astute investor would price the large standard error security at a lower price and a larger return would be expected. Observable market equilibrium prices and returns for securities and/or portfolios should reflect this additional risk component.

Assuming diversification of unsystematic risk this study hypothesizes that estimation risk regarding the beta estimate is priced in an equilibrium market. Specifically, this paper empirically tests for the presence of higher returns on portfolios which have a higher degree of estimation risk and lower returns on portfolios with a low degree of estimation risk.

Empirical Analysis

To test the hypothesis that portfolios with a higher degree of estimation risk, characterized by the standard error of the beta estimate, earn a higher return a buy and hold strategy of analysis was adopted. Three hundred and six firms listed on the New York Stock Exchange, excluding financial institutions and utilities, with calendar year ends were randomly chosen. The market model was estimated for each of these firms using OLS for the fifty-nine months beginning January 1975 and ending November 1979. The parameter estimates from this estimation was to develop high and low estimation risk portfolios. First, the standard errors of the beta coefficients were rank ordered. From this rank ordering, the sixty firms

with the largest standard errors for the beta coefficients and the sixty firms with the smallest standard errors for the beta coefficients were determined. The large standard error group and the small standard error group represented high estimation risk and low estimation risk groups of securities. From each of these two groups, twenty portfolios consisting of twenty randomly chosen securities were constructed. The beta estimates from the individual firm market model estimation were used to develop weights such that the beta for each of the portfolios was 1.00. Any portfolios which would require shortselling were eliminated and a new portfolio was formed. Only one portfolio from the high estimation risk group was eliminated while three portfolios from the low estimation risk group were dropped. The portfolios were weighted to provide a beta of 1.00 since Brown (1979) demonstrates that beta can be used to measure the degree of exposure to estimation risk. By forming all portfolios to have the same level of beta this variable was controlled and the exposure to estimation risk was held constant.

The weighting of each security in the portfolios was very straightforward since the overall beta of the portfolio is a linear combination of the securities in the portfolio. The magnitude of the estimation risk or standard error of the beta estimate for the portfolios is not as straightforward. The standard error of the beta estimates for the portfolios is a product of the variances of the returns for the individual firms and the covariances of the returns on the individual securities. In order to determine that the portfolios which were to represent high estimation risk and low estimation risk did possess differential levels regarding the standard errors for the estimated beta coefficients, the market model was estimated for each of the portfolios. Using the fifty-nine months from

January 1975 through November 1979, OLS estimates of the beta coefficients and the standard errors of the beta estimates were obtained. Table 1 provides the beta estimates and the standard errors for the portfolios of the high and low estimation risk groups. The standard errors of the beta estimates were different at the .001 level using a one-tailed t-test. This provided evidence that the two groups of portfolios were different in regards to the level of estimation risk.

[INSERT TABLE 1]

The hypothesis of this study predicts that the high estimation risk portfolios should out-perform the low estimation risk portfolios. The returns for the portfolios over a four month period were computed to assess any difference in returns for the two groups of portfolios. The cumulative return for each portfolio over the December 1979 through March 1980 period is presented in Table 2.

[INSERT TABLE 2]

The cumulative market return for the four month test period was -.06010. The low estimation risk portfolios had a mean standard error for the beta estimates of .0435 with a standard deviation of .0044. The high estimation risk portfolios had a mean standard error for the beta estimates of .0593 with a standard deviation of .0052. The cumulative returns for the low estimation risk portfolios ranged from -.139318 to -.050888 with a mean of -.0930 and standard deviation of .0266. For the high estimation risk group of portfolios the range of cumulative returns was -.104670 to .010639 with a

mean of -.0360 and a standard deviation of .0235. A one-tailed test of differences in the mean returns was conducted since the research hypothesis predicted that the returns of the high estimation risk portfolios should be greater than the returns of the low estimation risk portfolios. The results indicated that the null hypothesis of no differences should be rejected at the .001 level of significance. A nonparametric alternative, the Wilcoxin test for independent samples, provided similar results. Based upon these results, the evidence supported the hypothesis that estimation risk is priced by the market. The empirical results also indicate that neither of the groups of portfolios earned returns which were significantly different than the return on the market. However, the results do indicate that the return on the market is between the returns on the high estimation risk and the returns on the low estimation risk portfolios. Given that the market is made up of all securities, both high and low estimation risk firms, one would expect the market to earn a return that is greater than the portfolios with the least estimation risk and less than the portfolios with the largest estimation risk.

Summary and Implications

This study hypothesized that given all other things being equal, two securities or portfolios with differential levels of estimation risk would earn differential returns. Portfolios with different levels of estimation risk were constructed and the returns on the high and low estimation risk portfolios were compared. The empirical evidence upholds the hypothesis that estimation risk is priced in an equilibrium market.

The results of this study affect both practical and academic uses of the market model. From a practical point of view, portfolios or securities may possess the same estimate for the systematic risk coefficient but the uncertainty regarding the estimate does affect the overall risk of the

investment and the optimal portfolio choice should consider this additional risk component. In studies of information content and market efficiency portfolios or securities with the same beta estimate are treated as being equal when differential levels of beta reliability may exist. However, the magnitudes of the returns associated with these portfolios or securities can be linked to the uncertainty regarding the beta estimates (Ziebart, 1984).

Further research is needed to extend the generalizability of these results to other securities within the New York Stock Exchange as well as other exchanges. Also, other time periods should be tested to determine the stability of these results across different time periods. A further extension of this study could utilize weekly and/or daily returns for the analysis.

Table 1. Beta Estimates and Standard Errors for the Two Groups of Portfolios

	Low Estimation Risk Portfolios		High Estimation Risk Portfolios		
	Beta Estimate	Standard Error of Estimate		Beta Estimate	Standard Error of Estimate
1.	.998	.046	1.	.999	.055
2.	.999	.041	2.	.993	.062
3.	.996	.043	3.	1.000	.052
4.	.997	.045	4.	1.000	.065
5.	1.002	.042	5.	1.001	.067
6.	.998	.049	6.	.998	.065
7.	1.000	.044	7.	1.001	.055
8.	1.001	.042	8.	1.000	.058
9.	1.003	.036	9.	1.002	.063
10.	.999	.043	10.	1.002	.056
11.	1.002	.041	11.	1.001	.052
12.	1.000	.042	12.	1.001	.063
13.	1.002	.041	13.	.998	.059
14.	.999	.048	14.	1.002	.050
15.	.998	.048	15.	1.002	.058
16.	.998	.039	16.	1.002	.062
17.	.999	.035	17.	1.002	.068
18.	1.001	.050	18.	.998	.055
19.	1.000	.042	19.	1.000	.062
20.	1.000	.052	20.	.998	.059

Table 2. Cumulative Returns for the High and Low Estimation Risk Portfolios

Low Estimation Risk Portfolios		High Estimation Risk Portfolios	
#	Cumulative Return	#	Cumulative Return
1.	-.056842	1.	-.042322
2.	-.139318	2.	-.056929
3.	-.063301	3.	-.034370
4.	-.086522	4.	-.052919
5.	-.050888	5.	-.104670
6.	-.122164	6.	-.046668
7.	-.118508	7.	.010639
8.	-.075622	8.	-.028405
9.	-.115906	9.	-.008742
10.	-.099208	10.	-.029510
11.	-.096867	11.	-.025452
12.	-.066163	12.	-.025756
13.	-.073140	13.	-.048739
14.	-.061723	14.	-.022425
15.	-.119416	15.	-.012317
16.	-.103227	16.	-.020996
17.	-.117375	17.	-.049208
18.	-.109301	18.	-.046188
19.	-.116017	19.	-.027418
20.	-.067893	20.	-.048021

The market return for the four month period using the equally weighted index is -.06010